# Mathematics in Dialectics: Inquiries on Mathematical Process through Hegel, Marx and Lakatos

**Charles Jason P. Cahilig**
University of Santo Tomas | charlesjasonc@gmail.com

**Abstract:** It is arguable to conclude that mathematics is really consistent and adheres to a coherent system. This paper studies the extent of the dialectical process on the progress of mathematical knowledge. Individuals particularly mathematicians involved in an exchange and agreement of conceptual objectives require attention and recognition in the developmental process of mathematical notions. It is substantial to the approach of mathematicians and educators in mathematics to know what really their subject matter is. This paper aims to answer these questions: does the dialectical method a part of the nature of mathematics? If it does, what does it imply on the mathematical discoveries? Considering the dialectical process in mathematics, this aims to consider the rationale behind mathematical discoveries. The dialectical process provides a heuristic procedure that brings to the fundamentals of a proof in a theorem. The paper uses an archival approach to disclose the historical data needed for the dialectical nature of mathematics; the discussion among mathematicians through history provides significant indications on the growth of mathematical knowledge. The production of mathematical discoveries is a result of interactions of the mathematicians throughout history. The paper adopts Imre Lakatos' method of proofs and refutations and methodology of scientific research programmes to present the growth of mathematical discoveries with Hegel's and Marx's dialectics to support the foundation on the nature of mathematics as a dialectical. This paper will add to the answers on what mathematics is, what really its foundation and what is behind the certitude of mathematics. It can be used for future references to disclose theorems for the development on the discourse of mathematics.

**Keywords:** Lakatos, Mathematics, Dialectics, Marx, Hegel

Mathematics has a very significant role in justifying knowledge by providing a constructed model. It is in mathematics that the foundation of science is established; through the models, science is able to create statements from the truth that is



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claimed to be true.[1] The selection of model to affirm appropriately a scientific phenomenon render credits to mathematics in presenting models to be used. The rational systems and models presented by mathematics derived either from a heuristic process or by inductive or deductive reasoning. Through the collaboration of the inseparable qualities of science and mathematics led to knowledge or discoveries.

In every fortified system, there are foundations that support the structure. These foundations are the underlying elements which sustain the system. Foundations are the pillars which keep the erection of the structure. There is a careful statement of the fundamental generalities was given by Padoa in a paper before the Paris Congress of Philosophy, 1900:

> We have as a basal science logic, and as depending upon it the special deductive sciences which involve undefined symbols and whose propositions are not all capable of proof. The symbols denote either classes of 'elements or relations amongst elements. In any such science one may choose in various ways the system of un-defined symbols and the system of undemonstrated or primitive propositions, or postulates. Every proposition follows from the postulates by a finite number of logical steps.[2]

Relating it to mathematics thus it is founded by undefined symbols which represent abstract objects and by primitive propositions. [3] As Padoa stated, symbols denote elements that signifies descriptions to ensure the comprehension of the abstract entities of mathematics. Before a theorem was created, there are certain propositions that support a conclusion. These conclusions are accepted as true and precise because of its logical basis. Obtaining a definite system of undefined symbols and a definite system of postulates, it is necessary that it has a notion of the compatibility of these postulates.[4] Thus, it is subject to assessment; that is, that it is impossible to prove by a finite number of logical steps the simultaneous validity of a statement and its contradictory statement and Padoa introduces the idea of the irreducibility of the system of undefined symbols.[5] The idea of Padoa precludes reducing the primitive proposition into another proposition to retain its statement.

Mathematics is composed of proofs, conjectures and concepts which are elements of informal mathematics.[6] Yet these

[1] McClimans, Micah, *What is the difference between mathematics and science?* (February 3, 2015) https://www.quora.com/What-is-the-difference-between-mathematics-and-science, retrieved: April 25, 2017

[2] Moore, Eliakim Hastings. *On the Foundations of Mathematics*, "Science, New Series" Vol. 17, No. 428 (American Association for the Advancement of Science, 1903), p. 402.

[3] Mathematics is related to science. From the point of view of the scientist, mathematics is a tool of science; from the point of view of mathematicians, science is a field for the application of mathematical truth. For further readings, see Comstock, Clarence E., *Correlation of Mathematics and Science* "The Elementary School Teacher" Vol. 6, No. 2, 82 – 96.

[4] Moore, "On the Foundations of Mathematics," 402.

[5] *Ibid*., 402.

[6] Bart Van Kerkhove, "Aspects of Informal Mathematics: Towards a New Epistemology of

foundations was questioned by Lakatos and created a method to improve the mathematical knowledge. Concepts, conjectures and proofs develop as time passes by through continuous discussion for the sake of mathematical discovery. By the interactions of different mathematicians, they could discover or create a new idea which includes the methods of surrender, monster-barring, exception-barring, monster-adjusting, lemma-incorporation, and proofs and refutations.[7]

## GIST OF DIALECTICS IN MATHEMATICS

Axioms provide structural models – formal and informal mathematics – that can be applied to problems within mathematics as well as outside mathematics i.e. by application to architectural problems. However, dialectics argues that "none of these two fundamental positions alone can explain the nature of mathematics."[8] Models and structures do not consider the producing subjects as well as the concrete objects of production in a mathematical process.

The producing subjects or the mathematicians are responsible for the development of mathematics. They have the capacity to arrange, manipulate and create structures. Since they possess such capabilities, they are free to shape mathematics in any preference.

> Mathematicians are free to choose any consistent set of axioms they please. Since consistency is the only constraint, the choice must be a matter of subjective preference and is therefore philosophically uninteresting. […] They do not feel themselves to be constrained by consistency alone. When they evaluate a piece of research they are not satisfied with the information that all the theorems have been validly deduced from consistent premises. Some work is judged important, some trivial, some promising, some not, and so forth.[9]

Hence, mathematical process and development depends on the influence of the mathematicians. Consistency and accuracy are significant to mathematicians since it provides a firm structure or model that gives satisfaction to a rigorous development of mathematics; selective recognition and identification is the operation used to supply the necessary bits of information to the process.

The dialectical philosophy perceives human minds, however fallible, to be the only accessible and feasible medium for the greater rationality of mathematics.[10] The subject's rationale is the faculty of cognitive process that it possesses; the subject provides productivity and efficiency to constitute a structured model.

Mathematics," GAP.6-workshop, 14-16 September 2006, 4.

[7] Lakatos introduced these methods in his book *Proofs and Refutations: A Logic of Mathematical Discovery* whereas in an imaginary classroom setting, there are students conversing and arguing the definition of a complex polyhedron. For further readings about a computational model of Lakatos' theory, read Pease, Smail, Colton, Lee. *Bridging the gap between argumentation theory and the philosophy of mathematics* (Netherlands: Kluwer Academic Publishers, 2008), 5-6.

[8] Uwe V. Riss, "Objects and Processes in Mathematical Practice," *Foundations of science* 16, no. 4 (2011): 341.

[9] Brendan Larvor, "What is dialectical philosophy of mathematics?," *Philosophia Mathematica* 9, no. 2 (2001): 213.

[10] *Ibid*., 214.



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The dialectical nature of mathematics penetrates the reality or the principle behind the systematic structure of axioms. Considering dialectics as the nature of mathematics, Riss' states that: "Dialectics regards both subjective and objective capacities as inseparable moments of the mathematical development process. Each discovery depends on the subject's effort and capacity to find the solution as a moment of development; each invention also includes the aspect of *tripping over* the solution as a moment of discovery. The inventive character also comes to the fore in the fact that mathematicians often find several independent solutions to the same problem."[11]

## DIALECTICAL FRAMEWORK OF MATHEMATICS

Considering the history of mathematics, the notion of dialectics in the mathematical process is conducive for locating the changes in mathematical concepts. The development of mathematics, in a dialectical sense, offers contrasting concepts to engage in a discourse. The dispute between contrasting concepts will eventually be reconciled when the subjects arrive to a satisfactory conclusion. Moreover, dialectics and history are two sides of the same coin that are inseparable to each other.

> The objectivity of historical laws comes from the objectivity of human historical activity. Set apart from the objectivity of human historical activity – practice, the objectivity of historical laws would become a mystical force in control of human historical activity. Hegel's dialectics that "converges" with metaphysics describes the objectivity of law as the self-movement of "impersonal reason" and does not therefore mean an end to metaphysics but only its completion. Marx's dialectics that puts an end to metaphysics describes historical laws as "real people and their historical development" and does not therefore mean a convergence with metaphysics but an end to it.[12]

Hegel proposes a dialectical idealism however Marx, adopting Hegel's and modifying it in a material framework, introduces dialectical materialism. Hegel does not abandon metaphysics in his philosophy of history rather, his dialectics emphasizes the 'impersonal reason'. On the other hand, Marx deals with the real people who engage themselves to a discourse.

Sun made an evaluation of the contemporary significance of Hegel's dialectical metaphysics and he presents these four questions: "first, evaluation of the "history of metaphysics"; second, evaluation of Hegel's dialectics as the logic of the movement of human thought; third, evaluation of Hegel's dialectics as the logic of modernity; fourth, evaluation of the "true significance" of Hegel's dialectics for Marx's dialectics."[13] The historical significance of metaphysics portrays the development of a metaphysical thought. As the

[11] Riss, *op. cit.*, 346.

[12] Zhengyu Sun, "Dialectics: Hegel, Marx and post-metaphysics," *Social Sciences in China* 29, no.4 (2008): 16.

[13] *Ibid.*, 13.



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development of the metaphysical thought, it endures along with the movement of human thought through a dialectical logic. Marx adopts the dialectics of Hegel and he places it in a material conditions.

Moreover, in Hegel's words, he expresses that "the reals[14] themselves can be considered as examples of quantity but only as pure quantity. It may be interesting to remark that here, Hegel foreshadows the fact that if one generalizes (as Cantor did) the quantity of natural numbers as the cardinality of finite sets to the cardinality of the reals, one obtains a new concept not useful for measurement."[15] Real numbers were only deliberately argued and explained in the early 1860s, Hegel did not reach those years thus he uses neither that terminology nor particular mathematical designs and notation that were developed later. "But Hegel's 'quantity' and its associated logical categories represent another terminology for describing many of the mathematical features that since the 1860s have been taken to be essential to analysis and number theory and to the theory of the reals in particular."[16]

> Thus, Hegel's new argument here is not just that what mathematicians now call the real numbers are best understood against the background of our use of them in practices of measurement but rather that they can themselves be understood as measures, which is made explicit in the following section. That in terms of which we measure can be thought of as the relevant quality, even if that measure can itself be given a numerical form.[17]

On the other hand, Imre Lakatos who unconsciously followed Hegel's idea develop his historical observation into the historical conjecture that mathematical criticism was the motive behind the growth of mathematical knowledge during the nineteenth century. "More than thus, the philosophy of mathematics represented by P&R[18] also presents a rich analysis of how mathematics can be thought of as an essentially historical discipline, primarily through Lakatos' thoroughly brilliant and original historiography."[19] Kadvany argues that "somehow Lakatos has wedded his historicized philosophy of mathematics to Popper's falsificationist philosophy of "conjectures and refutations," after which P&R is self-consciously titled, but in spite of Popper's primary use of history as illustrative of methodological issues and not as a fundamental dimension of his logic of scientific discovery."[20] The dialectics is indeed present in the history of mathematics whereas conflicting ideas contend with each other.

> In brief, Lakatos argues that the usual formulation of refutation - the production of a counterexample to a

[14] Kaufmann and Yeomans refer it to mathematical objects i.e. rational or irrational numbers, geometrical figures, mathematical signs and symbols etc.

[15] Ralph Kaufmann and Christopher Yeomans, "Math by Pure Thinking: R First and the Divergence of Measures in Hegel's Philosophy of Mathematics," *European Journal of Philosophy* (2017): 5, doi:10.1111/ejop.12258.

[16] *Ibid., 2.*

[17] *Ibid*., 18.

[18] Lakatos' *Method of Proofs and Refutations*

[19] John Kadvany, "A Mathematical Bildungsroman," *History and Theory* 28, no. 1 (1989): 25.

[20] *Ibid*., 28.


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> theorem with respect to a fixed domain of mathematical objects -is insufficient to explain critical periods in the historical development of mathematics. The dialogue shows that in the creation of a mathematical falsification, counterexamples often become refutations only when accompanied by the informal expansion, or concept-stretching, of the concepts involved defining the theorem's domain. The counterexamples are formal, logical refutations in the expanded domain, perhaps also with an implied expansion to a new theoretical language, but the process of expansion itself is not part of the straightforward Popperian notion of falsification, which essentially depends only on classical logical contradiction.[21]

For both Hegel and Lakatos, knowledge reveals a certain improvement only if one's epistemic domain alters along with the criterion of truth. In Lakatos' case, Kadvany states: "the criterion is the proof, as it shows why the theorem is true, and his historical point about the method of proofs and refutations is that this pattern in the growth of mathematical knowledge is a relatively recent innovation. Lakatos, remarkably, has translated Hegel's phenomenological method not just into an account of nineteenth-century mathematics, but into a feature of mathematical criticism itself."[22]

Returning to Hegel, according to Sun, he included "the logical pre-existence of the identity of thought and being and the endogenous emergence of the difference between thought and being as a dual premise and attributed the possibility of cognition to the dialectical movement of concepts, that is, the unity of thought and being displayed as a process of the movement of concepts from abstract identity to concrete identity. […] the process of the identification of individual reason with universal reason is both a process of cognition from abstract identity to concrete identity and a process of freedom from abstract universality to concrete universality."[23]

> "History" as "the activity of man" is the form of men's being. The essential difference between man and animals is that man is a "historical" being. As "the first premise of all human history is, of course, the existence of living human individuals," Marx "set out from real, active men," from "the real individuals, their activity and the material conditions of their life." He said, "The existence of the human race is the result of an earlier process which organic life passed through. Man comes into existence only when a certain point is reached. But once man has emerged, he becomes the permanent pre-condition of human history, likewise its permanent product and result, and he is *pre-condition* only as his own product and result." Man himself acts as the "precondition" and "result" of

[21] *Ibid.*, 37.
[22] *Ibid.*, 38.
[23] Sun, *op. cit.,* 7.



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> history, and makes his "history" with his own activities and his "existence" with his own history. Without man's "history," man's "existence" and the relationship between man and the world would be abstract. The existence of the human race is the "actual life-process" of man; and the foundation of man's "actual life" is the production of means of material subsistence – labor. "Labor" is the "existence" of the human race.[24]

In his efforts to transform metaphysics through dialectics, Hegel, through his critique of the "abstract reason" that made up the old metaphysics, used the dialectical movement of concepts to realize a metaphysics in which thought defined concepts, unifying "total freedom" with the "necessity of links," and then reconstructed metaphysics into a dialectics in which ontology, epistemology and logic formed a unified whole. [25] The structure of this "union" is the arrangement of the important principles of the ultimate cause through the differences, inconsistencies and contradictions in concepts themselves from "abstract identity" (abstract universality) to "concrete identity" (concrete universality).

## THE *GEIST* AND MATHEMATICS

The dialectical scheme of Hegel is a necessary part of Lakatos's central thesis on mathematics. Though Hegel did not mention or disclose thoughts about mathematics in dialectics, it is evident however in his *The Science of Logic*, wherein he considers science following the dialectical process toward the *Geist*. However, Pinkard modified and reshaped Hegel's thoughts and established a philosophy of mathematics of Hegel. Pinkard observes that concepts are similar to chess pieces that imply normative character:

> Concepts may be thought of as such normative kinds. Their 'logical meaning' is that set of rules which constitute them, which in their case would be inference rules. The movement of concepts may be thus conceived in analogy to a game. The game analogy somewhat breaks down, however, when one asks what is moving. What moves in Hegel's theory would not really be the pieces, i.e., the concepts themselves, but thought itself. That is (to take the game analogy a bit further), each concept is a position in the game, and thought moves from one position to another. The meaning of all the particular concepts lies thus in where they are in the game.[26]

Furthermore, Hegel clarifies reason as negative and dialectical, "since it dissolves the determinations of the understanding into nothing; it is positive, since it generates the universal, and comprehends the particular therein. Just as the understanding is usually taken as something separate from reason in general, so also dialectical reason is taken as something separate from positive reason. In its truth reason is however spirit, which is higher than both reason bound to the understanding

[24] *Ibid.*, 10.
[25] *Ibid.,* 6.
[26] Pinkard, Hegel's Philosophy of Mathematics," *Philosophy and Phenomenological Research* 41, no.4 (1981): 455.




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and understanding bound to reason. It is the negative, that which constitutes the quality of both the dialectical reason and the understanding: it negates the simple, thereby posits the determinate difference of the understanding; but it equally dissolves this difference, and so it is dialectical."[27] It exhibits an internal revolution as the reason justifies it concurrently negates. Thus, there is an occurrence of construction of contradictions and adheres to the dialectical process.

In Hegel's view, inherent criticism and analysis delve into contradictions. It offers to disclose and dismiss the conflict between reality and its true essence. "So understood, however, immanent critique does not attend merely to discrepancies between reality and common conceptions concerning its essence. Its chief focus is not a tension between a thing and what conventionally passes as its true nature rather, its immanent critique examines essential contradictions, oppositions between a state of affairs and its true nature or essence."[28] In Hegel's terms, immanent critique recognizes self-contradictory, that which affirms then negates the affirmation i.e. contradicts its true being. A critique absorbing anything but essential self-contradiction does not analyze or consider the real in terms of its intrinsic logic but rather, purely abstract. It suggests an 'empty' notion; it expresses only the concept of an abstract *y* which is self-identical or a self-affirmation. Having laid this groundwork, Pinkard says that "Hegel goes on to speak of a plurality of such empty, self-identical units. He does not argue specifically for this move but justifies it by appeal to an earlier figure in the *Wissenchaft de Logik*, viz., the move from the concept of a determinate being to that of determinate beings. That transition was justified (roughly) by the claim that to say of anything that it is something or another is to contrast it with other different things."[29] Having done that Hegel continue to bring up by what means with the one is identical with the many, yet is not the same with the many.

Hegel attempts both to alter the way in which the objectivity of the results of "Spirit's experience to itself" is assessed and to defend such a newly defined objectivity. Pinkard asserts that "he (Hegel) tries to account for such things as how and why a subject would find its views of *another subject* "opposed" by such a subject; how social subjects, groups, or classes find their desires, and especially their view of their own desires, opposed and negated by other social subjects, groups, or classes; how political subjects with certain notions about political life would (and did) find themselves in sometimes "tragic" opposition; in what way laboring or even worshipping subjects find their experience if their own activity "in opposition with itself"; and so forth."[30] It stresses the manner of the subject that thinks of another subject as an opposing factor thus a self-contradictory illustration of the subject.

> Inasmuch as the new true object issues from it, this dialectical movement which consciousness

[27] Hegel, G.W.F., *The Science of Logic*, trans. and ed. George di Giovanni (Cambridge: Cambridge University Press, 2010), 10.

[28] Andrew Buchwalter, *Dialectics, Politics, and the Contemporary Value of Hegel's Practical Philosophy* (New York: Routledge, 2012), 47.

[29] Pinkard, *op. cit.*, 457.

[30] Robert Pippin, *Hegel's Idealism: The Satisfactions of Self-Consciousness* (Cambridge: Cambridge University Press, 1989), 101.



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> exercises on itself and which affects both its knowledge and its object, is precisely what is called *experience [Erfahrung]*. In this connection there is a moment in the process just mentioned which must be brought out more clearly, for through it a new light will be thrown on the exposition which follows. Consciousness knows *something;* this object is the essence or the *in-itself;* but it is also for consciousness the in-itself. This is where the ambiguity of this truth enters. We see that consciousness now has two objects: one is the first *in-itself*; the second is the *being-for-consciousness of this in-itself*. The latter appears at first sight to he merely the reflection of consciousness into itself, i.e. what consciousness has in mind is not an object, but only its knowledge of that first object. But, as was shown previously, the first object, in being known, is altered for consciousness; it ceases to be the in-itself, and becomes something that is the *in· itself* only *for consciousness.* And this then is the True: the being-for-consciousness of this in-itself. Or, in other words, this is the *essence,* or the *object* of consciousness. This new object contains the nothingness of the first, it is what experience has made of it.[31]

In Hegel's excerpt above in *Phenomenology of the Spirit*, he recognizes the dialectical method of the subject's inquiry on knowledge and its object of experience. He even expresses two objects of consciousness: in-itself, and the being-for-consciousness. "Experienced objects are always objects of judgments for Hegel, and although in successful knowledge such objects turn out to be "in themselves" as they are characterized "for consciousness," they are still taken to be so identical, in a mediated sense, as a result of the subject's establishing that this is so."[32]

> The progress made thus far in the dialectic of self-consciousness yields not just insight into the kind of object a subject must relate to in order to achieve satisfaction but also a conceptual revision of the ideal the subject aspires to realize. A crucial lesson that desire's experience has taught us is that true self-sufficiency for a subject – self-sufficiency that affords full and stable satisfaction – does not consist in absolute independence from everything other but involves instead dependence on other (numerically distinct) subjects that one also recognizes as in some sense oneself. In other words, the quest for recognition counts as a step forward for self-consciousness only if one assumes a corresponding revision in its understanding of what a subject's claim to be self-sufficient requires: a subject's dependence on another is compatible with a kind of self-sufficiency as long as that subject can see the being it depends on as sufficiently like itself (as of the same exalted type – subject rather than

[31] Hegel, *Phenomenology of the Spirit,* 55.

[32] Pippin, *op. cit.*, 114.

> mere thing – that it takes itself to be).[33]

Neuhousser affirms that dialectical process is a progression of the self-consciousness toward satisfaction. He exposes that Hegel extends the question of a historically proportionate or developmental assessment of the very basic principles to a "knowledge community" in which the subject comes to understand itself and the world. The subject's reliance on another thoroughly confirms or affirms the conscious subject's knowledge of the world. Hegel clears that "the dialectic of sense-certainty is nothing else but the simple history of its movement or of its experience and sense-certainty itself is nothing else but just this history. That is why the natural consciousness, too, is always reaching this result, learning from experience what is true in it; but equally it is always forgetting it and starting the movement all over again."[34] The self-affirmation and self-certainty of the subject is just but a historical process that persists to attain the end of the history.

The dialectic according to Hegel in his *The Science of Logic* states that "once considered a separate part of logic and, one may say, entirely misunderstood so far as its purpose and standpoint are concerned, and thereby assumes a totally different position."[35] In the *Phenomenology of the Spirit*, he affirms that "the singular being of sense does indeed vanish in the dialectical movement of immediate certainty and becomes universality, but it is only a *sensuous universality*. My 'meaning' has vanished, and perception takes the object as it is *in itself,* or as a universal as such. Singular being therefore emerges in the object as true singleness, as the in-itself of the One, or as a reflectedness-into-self."[36] As the dialectical process persists, the subject's own understanding disappears and shifts into the universal understanding.

> Dialectic is commonly regarded as an external and negative activity which does not belong to the fact itself but is rooted in mere conceit, in a subjective obsession for subverting and bringing to naught everything firm and true, or at least as in resulting in nothing but the vanity of the subject matter subjected to dialectical treatment. […] Because dialectic was held to be merely the art of practicing deceptions and producing illusions, it was straight away assumed that it plays a false game; that its whole power rests solely on hiding its deception; that its results are only deviously obtained, a subjective shine. […] It is in this dialectic as understood here and hence in grasping opposites in their unity, or the positive in the negative, that the speculative consists.[37]

Hegel never mentioned the "antithesis" rather he affirms that dialectic may appear and move into a negative one. "Dialectic as a negative movement, just as it immediately *is,* at first appears to consciousness

[33] Frederick Neuhouser, "Desire, Recognition, and the Relation between Bondsman," in *Hegel's Phenomenology of Spirit*, ed. Kenneth Westphal, 37 – 54 (Oxford: Blackwell Publishing Ltd, 2009), 46.

[34] Hegel, *Phenomenology of the Spirit,* 64.
[35] Hegel, *The Science of Logic*, 34.
[36] Hegel, *Phenomenology of the Spirit,* 77.
[37] Hegel, *The Science of Logic*, 35.



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as something which has it at its mercy, and which does not have its source in consciousness itself. […]Through this self-conscious negation it procures for its own self the certainty of its freedom, generates the experience of that freedom, and thereby raises it to truth."[38] As the negative movement of dialectic goes up against the opposites, it will eventually arrive to the truth, a "synthesis". Westphal contends that Hegel's notion of the dialectical contradictions or the negative movement of dialectics "neither denies nor violates the law of non-contradiction. Instead, Hegel holds that certain important truths can only (or at least can best) be expressed by using what appears to be a formal contradiction."[39]

According to Hegel, The *Geist* or the Thing itself unifies the opposites as the dialectic shift occurs. "By a new shift in the dialectic it is made plain that, if the Thing is conceived as absolutely One, it will be no longer possible to distinguish it from other things: all will be wholly blank unities and indistinguishable. A Thing must be what it is only by having its own properties, those proper and peculiar to itself. Since each of these properties has its own separate being, the Thing again becomes a loose association of properties: it is *A* and also B and also C, etc."[40] The Thing's properties either affirm or negate but what is definite and distinct constitute the totality of the Thing.

> An idealism which merely asserts this certainty (All the world is my idea) without going through the relevant dialectical preparation can neither explain nor understand itself. Its certainty always stands over against other certainties that the dialectical journey abolishes. The certainty of my rational ego always stands over against the certainty of something else existing alongside myself. The dialectical preparation establishes idealism as the only truth, but only in a general, abstract form which will have to be given concreteness in various actual sorts of confrontation.[41]

In *The Science of Logic*, Hegel adds that "the concept in its formal abstraction reveals itself to be incomplete and through a dialectic immanently grounded in it passes over into reality: it passes over into it, however, as into something which it generates out of itself, not as if it were falling back again onto a ready-made reality which it finds opposite it, or as if it were taking refuge, because it sought for something better but found none, into something that has already been proven to be the unessential element of appearance.[42] An abstracted concept is insufficient to reveal the truth rather it is necessary to undergone a dialectical process to disclose itself to the reality and to at least prove the unnecessary elements.

> The shape of the immediate concept constitutes the standpoint that makes of the concept a subjective thinking, a reflection external to the

[38] Hegel, *Phenomenology of the Spirit,* 124.

[39] Kenneth Westphal, "Hegel's Phenomenological Method and Analysis of Consciousness," in *Hegel's Phenomenology of Spirit*, ed. by Kenneth Westphal, 1 – 36 (Oxford: Blackwell Publishing Ltd, 2009), 14.

[40] Hegel, *Phenomenology of the Spirit,* 511.

[41] *Ibid.,* 528.

[42] Hegel, *The Science of Logic*, 522.



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> subject matter. This stage constitutes, therefore, subjectivity, or the formal concept. Its externality is manifested in the fixed being of its determinations that makes them come up each by itself, isolated and qualitative, and each only externally referred to the other. But the identity of the concept, which is precisely their inner or subjective essence, sets them in dialectical movement, and through this movement their singleness is sublated and with it also the separation of the concept from the subject matter, and what emerges as their truth is the totality which is the objective concept.[43]

Hegel, as influenced by Kant's noumenon and phenomenon, as stated from above, distinguishes the knowledge internal and external to the object. But he affirms that intrinsic to the object is what is set to a dialectical movement. Essentially, the truth that emanates from the separation of the concept from the subject matter is the entirety which is, according to Hegel, the objective concept. "This no less synthetic than analytic moment of the judgment through which the initial universal determines itself from within itself as the other of itself is to be called the dialectical moment. Dialectic is one of those ancient sciences that have been the most misjudged in the metaphysics of the moderns and in general also by popular philosophy, both ancient and recent. […]Dialectic has often been regarded as an art, as if it rested on a subjective talent and did not belong to the objectivity of the concept."[44]

> The fundamental prejudice here is that dialectic has only a negative result – a point about which more in a moment. First, regarding the said form in which dialectic usually makes its appearance, it is to be observed that according to that form the dialectic and its result affect a subject matter which is previously assumed or also the subjective cognition of it, and declare either the latter or the subject matter to be null and void, while, on the contrary, no attention is given to the determinations which are exhibited in the subject matter as in a third thing and presupposed as valid for themselves. […]In fact, therefore, everything rests on these determinations; they are the true subject matter and content of reason, and anything else that might be understood by subject matter and content in distinction from them has meaning only through them and in them. It must not therefore be taken as the fault of a subject matter or of the cognition that these determinations, because of what they are and the way they are externally joined, prove to be dialectical.[45]

Placing Hegel's dialectics in a mathematical setting, Sommers-Hall does it in calculus: "The dialectical approach to the calculus did not merely allow Hegel to propose a resolution to the problems of the calculus,

[43] *Ibid.,* 527.
[44] *Ibid.,* 741.
[45] *Ibid.,* 743.



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but also allowed Hegel to provide further support for his own dialectical method."[46] He adds: "The calculus allows us to characterize the relations between quantities whose ratios to one another vary. Although it can relate any variables, I will mostly refer to the relation between time and distance travelled by a hypothetical body."[47] Calculus simply deals with velocity; however, its approach extends to relations between quantities and ratios.

> The basic function of the differential calculus is to find the gradient of a curve at a particular point. The difficulty with a curve is that as the gradient varies, we need to find the gradient of a point. Gradient, however, seems to involve a difference. For Leibniz, at least in his informal account, the solution was to resort to the notion of the infinitesimal, as an infinitely small difference between two points, *dy*. As this difference was infinitely small, it could be discounted for the purposes of calculation, but, as it retained a magnitude relative to *dx*, it could be used to form a ratio, *dy*/*dx* which had a determinate value. […]Hegel rejects this Leibnizian approach since the idea of neglecting infinitesimal values, whilst it may lead us to the correct results, does not give us the rigour essential to a proper mathematical proof.[48]

On the other hand, Sommers-Hall continues that Newton's understanding of the calculus is integrated into Hegel's dialectic, as well as why Hegel thought a call for a dialectical foundation, "the idea of the ratio as it occurs in Hegel's remarks on the mathematical infinite in order to trace the genetic development of Hegel's own understanding of the ultimate ratio."[49] He states the in the differential relation, "we therefore have a situation whereby both the ratio itself as well as the terms can only be understood as a totality. For Hegel, this represents the true dialectical relation of the quantum and the infinite. In the differential ratio, each of the quanta can only exist in relation to the other. That is, the *dy* and *dx* only have meaning through their reciprocal relation."[50]

> The program involves a reconstruction according to this set of basic principles of the concepts of everyday experience (i.e., of the basic notions of those things with which we have an 'acquaintance,' Bekanntschaft),5 of the concepts of natural science,6 and of past philosophical theories. One of the basic aims in Hegel's program (although certainly not the only one) was the construction of what could be called a thoroughgoing compatibilist philosophy. That is, a basic tenet of Hegelianism (at least as Hegel saw it) was that many apparent contradictions in our categorical framework or many apparent

[46] Henry Somers-Hall, "Hegel and Deleuze on the metaphysical interpretation of the calculus," *Continental Philosophy Review* 42, no.4 (2010): 557.

[47] *Ibid.,* 557

[48] *Ibid.,* 560.

[49] *Ibid.,* 562.

[50] *Ibid.,* 563.



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> incompatibilities between competing categorical frameworks were only that: apparent and not real contradictions and incompatibilities. They could be 'overcome' (aufgehoben) if the conceptual framework was sufficiently expanded and ordered correctly.[51]

## DIALECTICAL MATERIALISM AND MATHEMATICS

Assuming that Hegel's conceptual dialectics is "full", "perfect", or "complete" metaphysics and the foundation for the idea of "truth" are and will continually be of self-sufficient and exceptional value and important in contemporary and future philosophical research and progress. Sun presents four questions that evaluates the contemporary significance of Hegel's dialectical metaphysics, "first, evaluation of the 'history of metaphysics'; second, evaluation of Hegel's dialectics as the logic of the movement of human thought; third, evaluation of Hegel's dialectics as the logic of modernity; fourth, evaluation of the 'true significance' of Hegel's dialectics for Marx's dialectics."[52]

Adopting and modifying Hegel's notion of the dialectical idealism, for Marx, according to Wood, "the dialectical structure of the world is a complex empirical fact about the nature of material reality. […] dialectical thinking only reflects the dialectical structure of the world which is thought about. If they think in terms of universal concepts, that is because it is the nature of thinking to abstract from particulars, and grasp their common properties."[53] He adds that: "Yet when Marx praises Hegel in such ways, he never fails to attack the 'mysticism' of Hegel's method or to emphasize that the Hegelian dialectic is not satisfactory as it is. […] Hegel's dialectic is, on the one hand, enclosed or shrouded in 'mysticism'. It must be 'stripped of' this false form, 'to discover the rational kernel in the mystical shelf'. On the other hand, with Hegel the dialectic is 'standing on its head'.[54]

In contrast with Hegel, Marx contends with idealism, rather, he deals with the materialism which reflects the structure of the world. Marx states that "The premises from which we start are not arbitrary; they are no dogmas but rather actual premises from which abstraction can be made only in imagination. They are the real individuals, their actions, and their material conditions of life, those which they find existing as well as those which they produce through their actions."[55]

Marx's dialectics that contradicts metaphysics portrays history as "real people and their historical development" and does not therefore convey a concurrence with metaphysics but a close encounter to it.

> The fact is, then, that definite individuals who are productively active in a specific way enter into these definite social and political relations. In each particular instance,

[51] Pinkard, *op cit.*, 453 – 454.

[52] Zhengyu Sun, "Dialectics: Hegel, Marx and post-metaphysics," *Social Sciences in China* 29, no. 4 (2008): 13.

[53] Allen Wood, *The Arguments of the Philosophers: Karl Marx* (London: Routledge, 1981), 209.

[54] *Ibid.*, 207.

[55] Karl Marx, "German Ideology," in *Karl Marx: Selected Readings*, ed. Lawrence H. Simon (Cambridge: Hackett Publishing Company, Inc., 1994), 107.



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> empirical observation must show empirically, without any mystification or speculation, the connection of social and political structure with production. The social structure and the state continually evolve out of life-process of definite individuals not as they may appear in their own or other people's imagination but rather as they *really* are, that is, as they work, produce materially, and act under definite material limitations, presuppositions, and conditions independent of their will.[56]

Sun affirms that Marx's dialectics is "by no means a dialectics that criticizes only "abstract reason," but also one that criticizes "abstract being" (capital), a dialectics whose criticism turns the independence and personality of capital into the independence and personality of man."[57]

"Dialectic is not a method in the sense of a set of rules or procedures for inquiry, or a general prolegomenon to science of the Baconian or Cartesian kind, which tries to prescribe the right way to employ our cognitive faculties irrespective of the way the objects of our knowledge may be constituted."[58] But dialectic does involve some recommendations about how science should approach the world, what sort of order to look for in it, what sorts of explanations to employ, even a theoretical program to be followed.

Marx states that in general view of the people "all collisions in history have their own origin in the contradiction between the productive forces and the form of interaction."[59] The origin of those contradictions varies from a particular historical setting. For Marx, according to Wood, "essential contradiction does not denote the discrepancy between an entity and a germinally present, albeit unrealized, conception of rationality; it designates the wholesale falsity of the real. This is the principle of negativity appropriate for a social theory determined to expose the falsity of a society based on the forcible exclusion of the producers from the means of production."[60] The contradictions alleviate the current paradigm for an improved model to conform to the changes. Marx sees the actual falsity of the society in which he realizes a need for reformation, a paradigm shift rather, to emancipate the oppressive system in the society.

The dialectics cannot be separated from a historical standpoint. It serves as the logic of history.

> (Marx) his dialectics was a dialectics concerned with "historical law." With Marx, "dialectics" does not negate "law," but is itself "law," or the "intensional logic of history" where ontology, epistemology and dialectics are unified with history as their common content. It is this dialectics that forms *Capital*, the "capitalized logic." In the sense that

[56] *Ibid.*, 111.
[57] Zhengyu Sun, *op.* cit., 11.
[58] Wood, *op. cit.*, 211.
[59] Marx, *German Ideology*, 142.
[60] Andrew Buchwalter, *Dialectics, Politics, and the Contemporary Value of Hegel's Practical Philosophy* (London: Routledge, 2012), 52.


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> Marx's dialectics is affirmed as the "intensional logic of history," "post-metaphysics" that "calls in question" the logic of truth-law-objectivity poses a challenge not only to Hegel's dialectics as the "intensional logic of thought," but also (or even more) to Marx's dialectics as the "intensional logic of history."[61]

Sun adds:

> The essential difference between the dialectics of Marx and that of Hegel lies in the question of whether real activities form laws or preexisting laws control activities. In essence, the "convergence" Hegel's dialectics that "converges" with metaphysics turns "laws" into a mystical force with "logical preexistence" and deduces history to be the self-fulfillment of logic. In contrast, Marx's dialectics that puts an end to metaphysics proceeds from human historical activity and not only "exposes the self-estrangement of human beings in their holy image," but also "exposes the self-estrangement of human beings in their unholy image," i.e., the self-estrangement of human beings in "capital," thus unifying human historical activity with historical laws. Therefore, the key to responding to the challenge posed by "post-metaphysics" to dialectics is to understand historical laws through human historical activity.[62]

Marx obtains Hegel's principle of the homogeneity of reason and reality when writing his own narrative of normative theory. "For Marx, Hegel's conjunction of thought and being furnishes the methodological parameters for a concept of social criticism that eschews the dichotomy of descriptive and prescriptive analysis. Marx calls this immanent critique."[63] Marx indicts Hegel's eradication of the line between the descriptive and prescriptive in social criticism thus it appears to have an endless critique which is inherently engraved in the methodological model.

Considering the philosophy of Marx, science was an empirical and theoretical activity conducted by a community of human inquirers. Observing it in a social setting, intersubjective collection of thoughts were gathered to create a particular affair of ideas, in other words, it supplies facts that, at various stages of scientific investigation through the cooperation of the scientific inquirers.[64] But Marx believed that progression in science is influenced by proliferation of theories, not merely by accumulation of facts. He also believed, according to Farr, "that the most important terms in theories could not be reduced to observational terms. Rather, they referred to entities, processes, or relations that were real causal agents in the world, the discovery and specification of which

[61] Sun, *op. cit.*, 16.
[62] *Ibid.*, 16.
[63] Buchwalter, *op. cit.*, 41.
[64] James Farr, "Science: Realism, criticism, history," in *The Cambridge Companion to Marx*, ed. Terrell Carver (Cambridge: Cambridge University Press, 1991), 109.

comprised the heart of the theoretical enterprise."[65]

> […] immanent critique evaluates reality not with alien principles of rationality but those intrinsic to reality itself. An immanent approach to social criticism exposes the way reality conflicts not with some "transcendent" concept of rationality but with its own avowed norms. In Hegelian speculation, Marx finds the rudiments of an objective or "scientific" approach to critical theory, one in which reality is challenged not with arbitrary constructions but with norms whose acknowledged validity is part and parcel of social reality itself.[66]

Through concrete activity and processing, the action goal of the subject is realized; "they must grasp, transform, and combine the selected objects. In this way they transcend the consideration of isolated objects and include the opposition that the setting activity has set aside. The concrete act actualizes the subjective as well as the objective capacities."[67] In other words, the accomplishment of the action can only assume its success if the subject possesses the requisite capabilities and the objects possess the appropriate properties.

According to Riss, dialectics "regards both subjective and objective capacities as inseparable moments of the mathematical development process. Each discovery depends on the subject's effort and capacity to find the solution as a moment of development; each invention also includes the aspect of *tripping over* the solution as a moment of discovery."[68] The innovative character also reaches the fact that mathematicians generally get several individual solutions to the same problem.

In the philosophy of mathematics, one would find the question of the nature of the existence of mathematical entities; whether it exists inside or outside the mind. Considering the realism of mathematical objects, acquiring the knowledge of them comes into place or rather they are invented or discovered and how do they grow and develop. Mathematics is known for its consistency and precision for truth and validity; thus mathematicians try to avoid contradiction that will affect the credibility of mathematics. Though getting not much attention from most of the people that Marx showed an interest to mathematics, the *Mathematical Manuscript* contains Marx's analysis on the foundation of calculus emphasizing the differential calculus. The aim of Marx is to expose the underlying reality behind the emergence of mathematical knowledge. Karl Marx proposes the dialectics of mathematics; dialectics, according to Blunden, is a science of contradiction which is being applied to mathematics to show its development. He adds that dialectics is also dialectic logic, a guide which indicates a path for the deduction of new, more concrete, truths.[69]

[65] *Ibid.*, 110.

[66] Buchwalter, *op. cit.*, 42.

[67] Uwe V. Riss, "Objects and Processes in Mathematical Practice," *Foundations of Science* 16, no. 4 (2011):342.

[68] *Ibid.*, *346*.

[69] *Cf.* Andy Blunden, "Dialectics and Mathematics," *Labour Review* (1984), http://ethicalpolitics.org/ablunden/pdfs/Dialectics_Mathematics.pdf


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The central topic of Carchedi's argument revolves on the comparison of Marx's method of finding the derivative of a simple algebraic function with an older, Leibniz-like method for finding the derivative of the same function.[70] Carchedi uses an example, the function $y = x^3$. The method of Leibniz on finding the derivative of this function, it starts with $dx = x_1 - x$ and $dy = y_1 - y$. That is, $(x_1, y_1)$ is a point of the function some distance from an arbitrary point $(x, y)$ and $dx$ are the distances in the $x$ and $y$ directions, respectively, between $x$ and $x_1$, and between $y$ and $y_1$, respectively. It results that $x_1 = x + dx$ and $y_1 = y + dy$. Marx felt that a more principled method for finding derivatives should be developed. For Marx, differential calculus can be used to compute the instantaneous rate of change in the profit rate. Marx's method of differentiation provides key insights into what was Marx's dialectical view of reality. Marx's method of differentiation reflects "a process that is real, temporal, in which something (a real number for example) cannot be at the same time also something else (zero) and in which movement affects the whole rather than only a part of it and is the result of the interplay of potentials and realized."[71]

Mathematics must be allowed to speak to us through its history and not act as a screen on which to project our philosophical or sociological fantasies.[72] This is what Lakatos proposed in a mathematical process. Returning to the history of mathematics, it enables to observe the feasible needs for clarification and verification of such discoveries whether it is still accepted for the time being. History can show the origin of the established characteristics of mathematics and analyzes its foundations. According to Dusek, Lakatos and Marx share the same thought:

> Lakatos' use of rational reconstruction in his account of history has a strong resemblance to that of Marx's treatment of economic history. Marx presents an account of economic development, but it is not a simple narration of history. Rather Marx has a schematic model, which does not fit exactly all the vagaries of history. He then adds successive complications, particularly in volume III of Capital (Marx 1967), which brings the model closer to the actual structure of capitalism. Nevertheless, the resulting corrected model remains a model, not a literal, detailed historical description capitalism. Lakatos, in his rational reconstructions, similarly does not simply present historical narration. Similarly Lakatos claims that his concept of the mathematician as imperfect personification of mathematics is "closely analogous to Marx's concept of the capitalist as the

[70] Russell Dale, "Guglielmo Carchedi on Marx, Calculus, Time and Dialectics," *Science & Society* 75, no. 4 (2011): 556.

[71] Guglielmo Carchedi, "Dialectics and Temporality in Marx's Mathematical Manuscripts," *Science & Society* 72, no. 4 (2008): 424.

[72] David Corfield, "Argumentation and the Mathematical Process," in *Appraising Lakatos: Mathematics, Methodology and the Man*, eds. George Kampis, Ladislav Kvasz and Michael Stöltzner (Netherlands: Kluwer Academic Publishers, 2002), 130.



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> imperfect personification of Capital" (Lakatos 1976, 189n1).[73]

Marx's materialistic history pervades the reconstruction of the society by emancipation from the oppression of the structure. Lakatos, on the other hand, is reinterpreted in the context of mathematics. Time will tell if a particular theorem is refuted and needed to be reconstructed. Moreover, Lakatos claims that the relation between historical theses and historical texts in mathematics is detailed and shows the relation between physical theories and their empirical evidence.[74] On this evidence it would appear that Lakatos' mathematical dialectic is purely epistemological. For him, mathematical knowledge grows and reinvents itself in order to improve our apprehension of a fixed mathematical reality.

> As the result of the process, new objects come into being. In materialistic dialectics the term *sublation* describes the negation of objectivity of the original objects in the processing activity that is followed by the negation of this process in re-establishing objectivity in its results. […]The particularly materialistic aspect in this consideration is the idea that sublation is not a mere intellectual act but concretized in material transformation.[75]

Marx asserts that the theoretical notion of the relationship of reason and reality hinders Hegel from extricating existence and actuality in an adversely significant way. "Actualization denotes not the process in which rationality only implicitly present in existing reality attains true being, but that through which a metaphysical concept of reason finds the external expression required for its complete realization."[76] Thus, the progression from partial to complete reality only expresses the absolute movement of a materialized subject, a process that leaves existing reality intact.

## CONCLUSION: LAKATOS' DIALECTICS IN MATHEMATICS

Hegel considers the goal of philosophy as directing the "rule of Reason" in the "unfolding of Spirit" through world history. Hegel's logic can thus well be corrupted or rather manipulated to justify anything that happens in history. Motterlini states that the adverse indication is that "whatever has been successful is thereby also somehow "right" and superior to what has been unsuccessful. The dangers of the dialectical approach lie obviously in the *authoritarian* attitude and in the *inexorability* of the process as formulated by Hegel first and then by Marx. […]*In sum,* Lakatos made use of Popper's fallibilism to wring authoritarianism out from Hegel's dialectical process and made use of the Hegelian idea of a dynamically unfolding rationality underlying the growth of knowledge to reject the alleged irrationality of the context of discovery."[77]Lakatos made use of dialectics

[73] Val Dusek, "Lakatos between Marxism and the Hungarian heuristic tradition," *Stud East Eur Thought* 67 (2015): 64.

[74] Brendan Larvor, "Lakatos as Historian of Mathematics," *Philosophia Mathematica* 3, no.3 (1997): 60

[75] Riss, *op. cit.*, 343.

[76] Buchwalter, *op. cit.*, 44.

[77] Matteo Motterlini, "Professor Lakatos Between the Hegelian Devil and the Popperian Deep Blue Sea," in *Appraising Lakatos: Mathematics, Methodology*



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to suffice the constant discovery of new programmes.

Besides from Popperian falsification to explain the rationality of scientific discovery, Lakatos adopts the Kuhnian paradigm and modifies it; he called it research programme.

> For in large research programmes there are always known anomalies: normally the researcher puts them aside and follows the positive heuristic of the programme.' In general he rivets his attention on the positive heuristic rather than on the distracting anomalies, and hopes that the 'recalcitrant instances' will be turned into confirming instances as the programme progresses. On Popper's terms the greatest scientists in these situations used forbidden gambits, *ad hoc* stratagems: instead of regarding Mercury's anomalous perihelion as a falsification of the Newtonian theory of our planetary system and thus as a reason for its rejection, most physicists shelved it as a problematic instance to be solved at some later stage - or offered ad hoc solutions. This methodological attitude of treating as (mere) anomalies what Popper would regard as (dramatic) counterexamples is commonly accepted by the best scientists. Some of the research programmes now held in highest esteem by the scientific community progressed in an ocean of anomalies. That in their choice of problems the greatest scientists 'uncritically' ignore anomalies (and that they isolate them with the help of ad *hoc* stratagems) offers, at least on our meta-criterion, a further falsification of Popper's methodology.[78]

Before dealing with the philosophy of mathematics of Lakatos, it is advantageously start with the methodology of scientific research programme. Paradigm of Kuhn and the programme share the same characteristic; they are both disturbed by anomalies. By means of anomalies, rational reconstructions occur to repair the vulnerable points of the paradigm. However, in research programme, the protective belt is the part being falsified; the hard core remains. All the rational reconstructions of history force history of science to embrace hypocritical morality, thus creating complicated histories, which depend on mythical 'inductive bases ', ' valid inductive generalizations ', 'crucial experiments ', 'great revolutionary simplifications' *et. al.*

> While maintaining that a theory of rationality has to try to organize basic value judgments in universal, coherent frameworks, we do not have to reject such a framework immediately merely because of some anomalies or other inconsistencies. We should, of course, insist that a good rationality theory must anticipate further basic value judgments unexpected in the light of

*and the Man*, ed. George Kampis, Ladislav Kvasz, and Michael Stöltzner (Dordrecht: Springer Science+Business Media, 2002), 29.

[78] Imre Lakatos, *Methodology of Scientific Research Programmes: Philosophical Papers Volume 1*, ed. John Worrall and Gregory Currie (Cambridge: Cambridge University Press, 1978), 126.




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> its predecessors or that it must even lead to the revision of previously held basic value judgments.' We then reject a rationality theory only for a better one, for one which, in this 'quasi-empirical' sense, represents a progressive *shift* in the sequence of research programmes of rational reconstructions. Thus this new - more lenient - meta-criterion enables us to compare rival logics of discovery and discern growth in 'meta-scientific' - methodological - knowledge.[79]

Paradigm is immediately denied and overthrown as soon as anomalies display the loop holes or the crisis in the current paradigm; thus new and modified paradigm emerges. In contrast with Kuhn's paradigm, Lakatos's research programme is not rejected instantly because of inconsistencies or anomalies. Methodology of Scientific Research Programme identifies anomalies in the framework of research programme; it appears that it follows an empirical approach in accomplishing a shift from the inadequate programme to a new one. However, the protective belt is where the falsification happens. The hard core retains the identity and contains the fundamental postulates of the programme; they are retained during the program's lifetime. However, protective belt is the modification of the programme.

> Indeed, this methodology confidently predicts that where the falsificationist sees the instant defeat of a theory through a simple battle with some fact, the historian will detect a complicated war of attrition, starting long before, and ending after, the alleged 'brucial experiment'; and where the falsificationist sees consistent and unrefuted theories, it predicts the existence of hordes of known anomalies in research programmes progressing on possibly inconsistent foundations. Where the conventionalist sees the clue to the victory of a theory over its predecessor in the former's intuitive simplicity, this methodology predicts that it will be found that victory was due to empirical degeneration in the old and empirical progress in the new programmes. […]Thus *progress in the theory of scientific rationality is marked by discoveries of novel historical facts, by the reconstruction of a growing bulk of value-impregnated history* as *rational.*[80]

Through discoveries of factual narratives which were believed and fortified by history, scientific rationality is justified.

On the other hand, Lakatos's philosophy of mathematics progressively separates *reliability* from *certainty* in mathematics. Assuming the proof of a theorem in a given axiomatic-formal system; "if it is accepted that the latter is consistent, it could thereby exclude the possibility of formalizing any counter-example in terms of the given system. But mathematics in the *making,* mathematics in its *growing process,* rarely expresses itself in axiomatic-formal theories; instead, mathematicians too make progress through conjectures, *experiments* and refutations. […] Lakatos considers *informal proof*

[79] *Ibid.*, 132.

[80] *Ibid.*, 133.



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as just another name for *thought experiment*. […] so *in mathematics* we have to analyse "proof-thought experiments" in order to find the hidden assumption from which a paradoxical result or contradiction follows."[81] It is not always simple to attend to an anomaly of a scientific theory; likewise, it is not always simple to attend to a counterexample in mathematics. The proof-thought experiment is the outset to prove the conjecture, however, it does not prove rather it falsifies.

Method of Proofs and Refutations has character of a heuristic, therefore it concerns with the autonomous dialectic of mathematics. Furthermore, for Lakatos, Euclidianism and formalism support a fixed conception of rationality. Heuristic, on the contrary, relates to the dynamic movement of concepts. Motterlini illustrates the Hegelian dialectical triad lurks behind the historical reconstruction of Euler's theorem:[82]

> THESIS: Primitive conjecture
>
> ANTITHESIS: Counterexample
>
> SYNTHESIS: Theorem and proof generated concept (+ lemma incorporation)

The heuristic rules of *Proofs and Refutations,* on the other hand, belong to the realm of the objective growth of knowledge, in a Popperian terminology, in the sense of world 3 rather than to mathematicians' minds in the sense of world 2.

It remains valid for mathematics too that one should examine a series of propositions rather than a single one. Motterlini explains in detailed the method of proofs and refutations:

> Since 1961 , Lakatos had claimed that when handling a counter-example to the initial conjecture ("global counter-example") or to any of the lemmas ("local counter-examples") one has to *choose* between various lines of behaviour each characterised by promises and risks peculiar to itself. The main case study of *Proofs and Refutations* illustrates the different possible strategies available in dealing with polyhedra that are exceptions to Euler's conjecture. A possible choice involves "eliminating monstrosities" by refining the concept of polyhedron; another consists in "surrendering to the counterexamples" thereby declaring the conjecture to be false; yet another in making certain " hidden lemmas" from background knowledge explicit and inserting them into the theorem enunciation, etc. The first kind of tactic reduces the content of Euler's conjecture and turns it into a " miserable convention." The second move is that of "naive" falsificationism in mathematics. The point is, of course, that one must recognise the element inherent of decision both in *restricting* the meaning of basic terms (such as polyhedron, vertex, edge, face etc.)

[81] Matteo Motterlini, "Professor Lakatos Between the Hegelian Devil and the Popperian Deep Blue Sea," in *Appraising Lakatos: Mathematics, Methodology and the Man*, ed. George Kampis, Ladislav Kvasz, and Michael Stöltzner (Dordrecht: Springer Science+Business Media, 2002), 27.

[82] *Ibid*., 28.



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> and in *extending* it. There is growth of mathematical knowledge in those same "adjustments" which allow what in the empirical sciences is an increase in (corroborated) content. I.e. those "stratagems" which allow one to invent/discover new problems, solve some of them and give a more elegant formulation of a simple conjunction of restrictive clauses. Only this third way corresponds to the "sophisticated falsificationism" in the empirical sciences.[83]

Given the illustration above, it may assume that mathematics is a product of human activity that 'alienates itself,' it is autonomous and objective. Thus the intention of heuristic methodology is to accept the logic of the development of mathematics, the dialectical method of growth, the incomplete rationality of mathematics.[84] "From this point of view, the heuristic-methodology looks *backward* to identify the rules that made such a growth possible in the past, and at the same time it looks *forward* to advice on how to obtain progress in the future. "Heuristic-methodology, although fallible, is both evaluative and normative. Generally speaking, from the Hegelian-Marxist view the aim of philosophy is not a contemplation of eternal truths, but rather an effort to interpret the present in the light of the past with a view to shaping the present for a better (utopian) future. Thus, for the Hegelian philosopher of mathematics, methodology, heuristic and logic of discovery are synonymous."[85] As assumed, Lakatos is inclined with Hegelian-Marxist methodology evident in his method of proofs and refutations. Heuristic concerns the set of rules and strategies to be adopted in the context of discovery but heuristic principles are not objective and autonomous.

Returning from Lakatos' philosophy of science, the basic unit of scientific progress is the "research program." Science is considered to be an ongoing contest among research programs. "The anatomy of a research program is characterized by two elements, namely, its "hard core" and its "positive heuristic". The positive heuristic spells out guidelines for the program's future theoretical development. It singles out significant problems, and sketches pathways to their solution. Thus, it directs scientists through the maze of confusing difficulties by giving them an idea where to drive at. The positive heuristic draws a picture of the program's "better self." One of Lakatos' historical assertions is that the development of a qualified program is determined by its heuristic."[86]

Kuhn's claims with respect to paradigm immunity and paradigm substitution are leveled as descriptive objections at methodological requirements inspired by Popper. Carrier states that:

> They are advanced as historical counterexamples to Popper's demands. It is true: a historical description cannot refute a

[83] *Ibid.*, 31.
[84] *Ibid.*, 34.
[85] *Ibid.*, 34.
[86] Martin Carrier, "Explaining Scientific Progress: Lakatos' Methodological Account of Kuhnian Patterns of Theory Change." In *Appraising Lakatos: Mathematics, Methodology and the Man*, ed. George Kampis, Ladislav Kvasz and Michael Stöltzner (Dordrecht: Springer Science+Business Media, 2002), 60.



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> methodological norm. But Popper and Kuhn share the commitment that historical theory change, at least concerning its fundamental and long-term aspects, is rationally justified and constitutes growth of knowledge. In light of this commitment, historical arguments cannot be dismissed offhand. Within the framework of Lakatos' methodology, by contrast, the two features of immunity and substitution constitute theorems rather than objections. They follow from Lakatos' conception of how theories are to be evaluated. The important result is that if theory change occurs the way Kuhn says, it does not violate methodological rules but, rather, conforms to such rules. Lakatos' conception is thus able to provide a methodological explanation of these Kuhnian characteristics of scientific change.[87]

Popper, Kuhn and Lakatos share similarities in their scientific inquiries about the growth of scientific knowledge. Popper introduces the theory of falsification i.e. theories are falsified by counterexamples precisely to 'prove', in a way, the conjecture. Kuhn proposes to look at the behavior of the scientific community. Social behavior of the scientific community affects the movement of paradigm in the context of normal science. Lakatos improves them; Popper denies the inductive reasoning through sense-experience and Kuhn's rejection of a methodological process of accumulation of scientific knowledge. Lakatos reconcile the two whereas anomalies happen, that cannot be denied, and in time of crisis, that is inadequate to Kuhn, falsification is expressed to improve the protective belt.

> Dialectics (in its explicit form, as we encounter it in Hegel or Marxism, as well as in its implicit form , as it is present in the work of Lakatos or Popper) has a tendency to interpret the evolution of concepts, or of knowledge in general, as a process which is logical in nature. It is not by chance that Hegel called dialectics a *science of logic,* and it is not by chance that Popper speaks about the *logic of scientific discovery*. According to dialecticians, knowledge is logical in nature. A necessary consequence of this is that the evolution of knowledge comes into conflict with logic.[88]

It is undeniable to claim that dialectics is present in Lakatosian philosophy of mathematics supported by Popper and Kuhn. The fact that Lakatos did not reconstruct any deeper conceptual change in history of mathematics or physics is not by chance. "As a dialectician, he conceived evolution to be in conflict with logic, but as Popper's disciple he was not prepared to give up logic. Thus he omitted some of the most interesting moments in the history of mathematics. If he had tried to reconstruct them, he would have been forced to violate logic. Therefore he

[87] *Ibid.*, 65.

[88] Ladislav Kvasz, "Lakatos' Methodology Between Logic and Dialectic," in *Appraising Lakatos: Mathematics, Methodology and the Man*, ed. George Kampis, Ladislav Kvasz and Michael Stöltzner (Dordrecht: Springer Science+Business Media, 2002), 229.



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reconstructed only those changes, in which relatively small conceptual changes occur."[89]

That is to say, history is the court in which philosophical accounts of mathematics are to be judged. This is not simply a question of laying a philosophically-generated narrative alongside a body of incorrigible historical evidence, just as the appraisal of a physical theory is not a matter of comparing it with an incorrigible empirical given. Indeed, we should expect a truly excellent philosophy of mathematics to change our view of the history of mathematics.[90] Larvor, indeed, affirms the dialectics in Lakatosian method. He adds that Lakatos "distinguishes between 'logical' refutations (where a theory is confronted by a piece of evidence with which it is strictly inconsistent) and 'heuristic' refutations. In a heuristic refutation, a theory is shown to be insufficiently general, or degenerative, or unable to survive translation into a vocabulary with greater explanatory power."[91] Recalling Lakatos's fallibilism aggregates the claim in which explanatory theories in mathematics discards the rational grounds in favor of their rivals. Thus, various interpretations and explanations are open for rationalization which could be rigorously selected by consensus.

[89] *Ibid*., 229.

[90] Brendan P. Larvor, "Lakatos as historian of mathematics," *Philosophia Mathematica* 5, no.1 (1997): 42.

[91] *Ibid*., 44.



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